Information Scrambling and Recovery in Inhomogeneous Quenches: An Exploration in 2d CFTs

Mao Tian Tan

Asia Pacific Center for Theoretical Physics

Based on arXiv: 2112.14388 and arXiv: 2302.08009 with Kanato Goto (YITP / Princeton), Masahiro Nozaki (KITS, UCAS), Kotaro Tamaoka (Nihon Univ.) and Shinsei Ryu (Princeton)

Inhomogeneous Quench in 1+1d CFT

- Analytically tractable models of non-equilibrium dynamics
- Consider 2d Free Fermion CFT (integrable) and holographic CFTs (chaotic)
- First Part: Inhomogeneous Quench of Thermal State
- Second Part: Information scrambling of Inhomogeneous Quenches.

Inhomogeneous Quench in 1+1d CFT

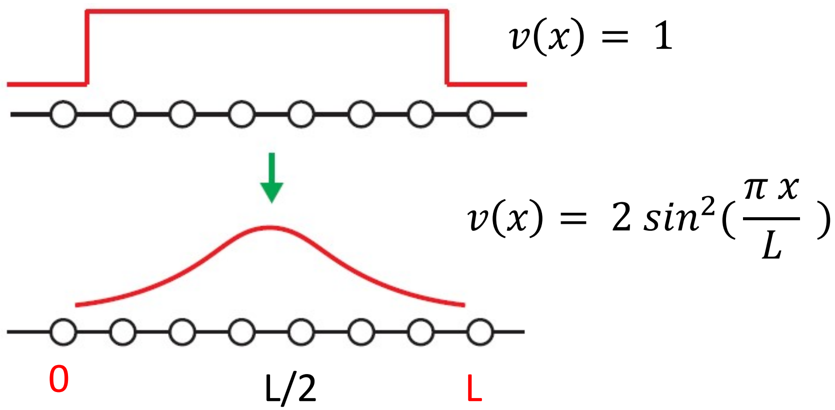
- Let $h(x)$ be the energy density so that

$$H_0 = \int_0^L dx\, h(x)$$

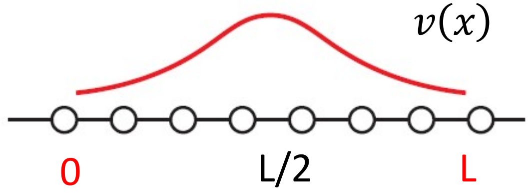
- The spatially inhomogeneous sine-squared deformed (SSD) Hamiltonian:

$$H_{SSD} = \int_0^L dx\, 2\sin^2\left(\frac{\pi x}{L}\right)\ h(x)$$

- SSD envelope vanishes at 0 and maximum at $\frac{L}{2}$



Quench of Thermal State



- Quench the uniform thermal state with inhomogeneous Hamiltonian in 1+1d CFT

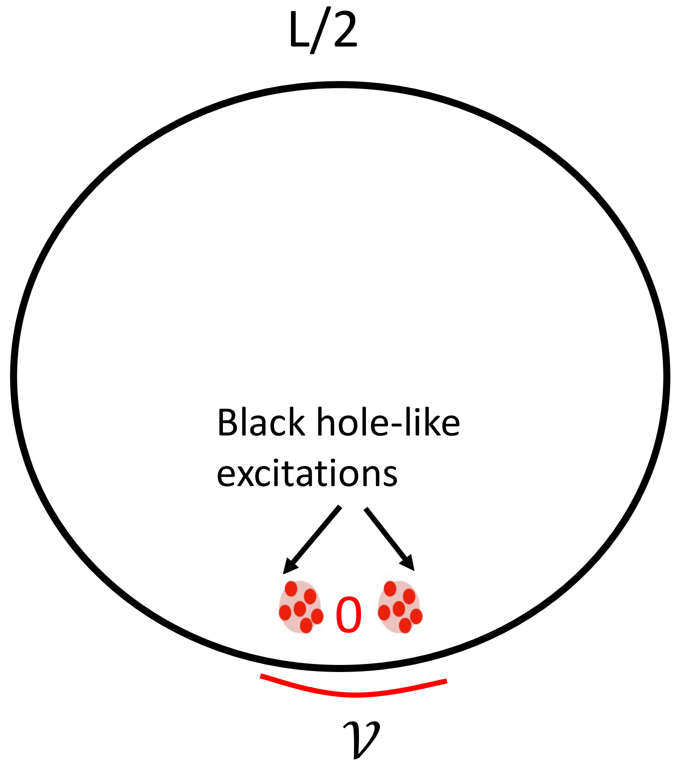
$$\rho(t) = e^{-iH_{SSD}t}\,\frac{e^{-\beta H_0}}{Z}\,e^{iH_{SSD}t}$$

- At late times

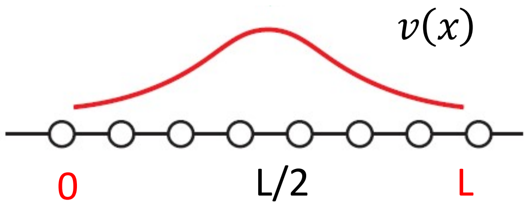
$$\rho \approx \rho_{\mathcal{V}} \otimes Tr_{\mathcal{V}}(|0\rangle\langle 0|)$$

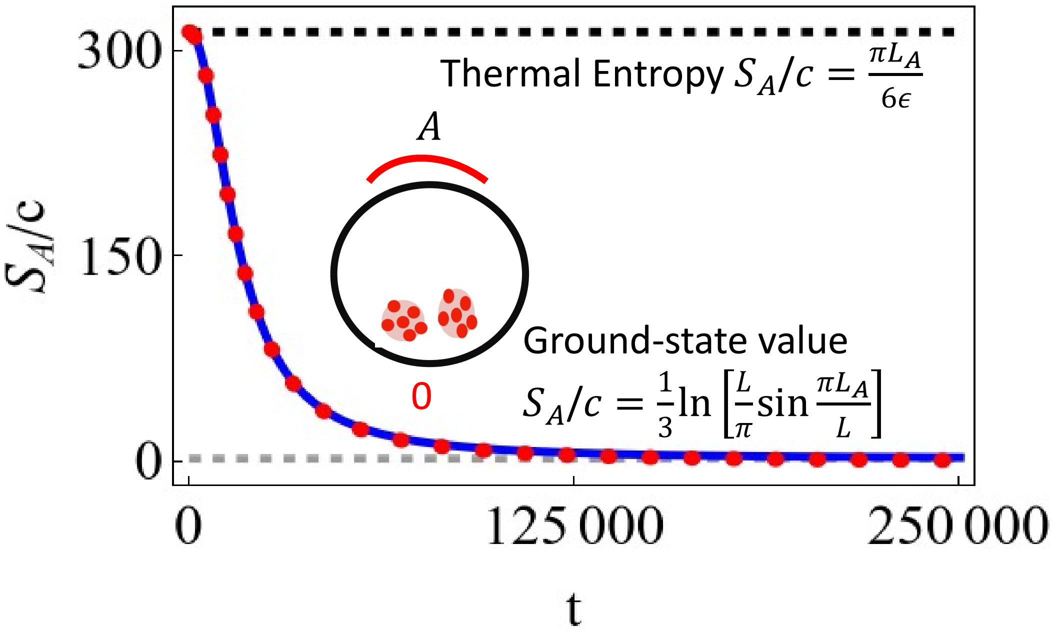
where $\mathcal{V}$ is a small subsystem that includes the origin,

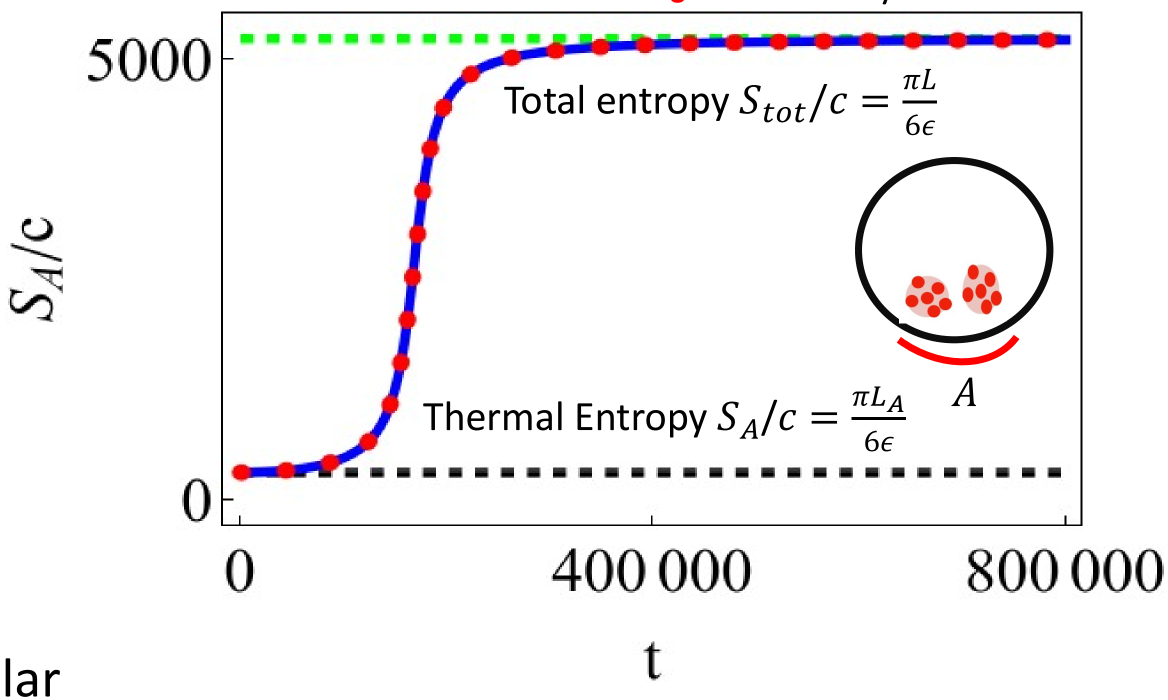
- Away from origin, cooled to ground state
- "Black hole-like" excitations at the origin that carry the total thermal entropy



Entanglement Entropy during Quench







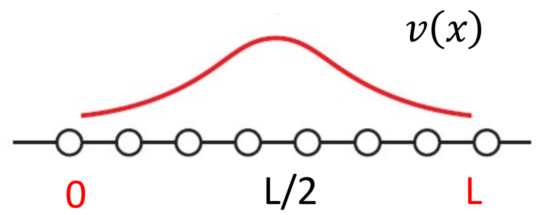
- Holographic CFT and free fermion CFT similar
- When subsystem is away from origin, entanglement entropy goes from thermal entropy of subsystem to ground state entanglement entropy
- When subsystem contains the origin, entanglement entropy goes from thermal value to thermal entropy of the total system

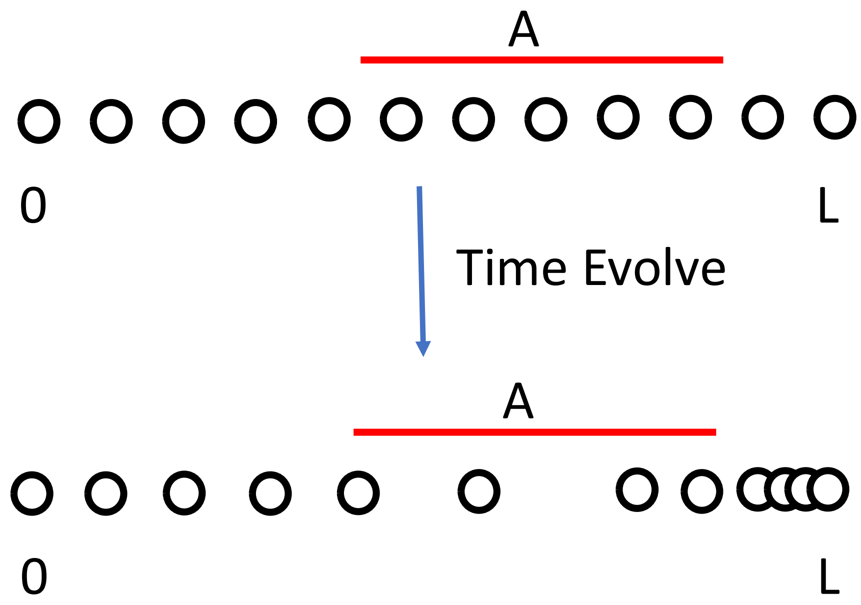
Quasiparticle Picture For Free Fermions

- Quasiparticle = quanta of information
- Thermal State = Uniformly distributed quasiparticles
- Half are left-moving, half are right-moving
- Inhomogeneous Quench ⇒ Quasiparticles move with spatially dependent speed

$$\mathrm{v(x)} = 2\sin^2\left(\frac{\pi x}{L}\right)$$

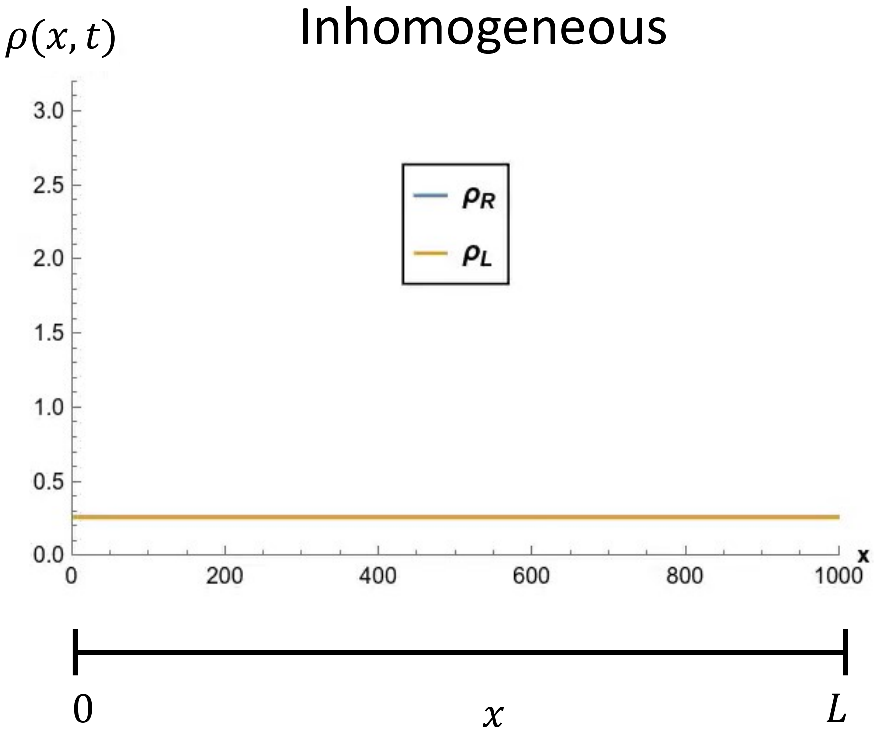
- Entanglement Entropy ~ No. of quasiparticles in A

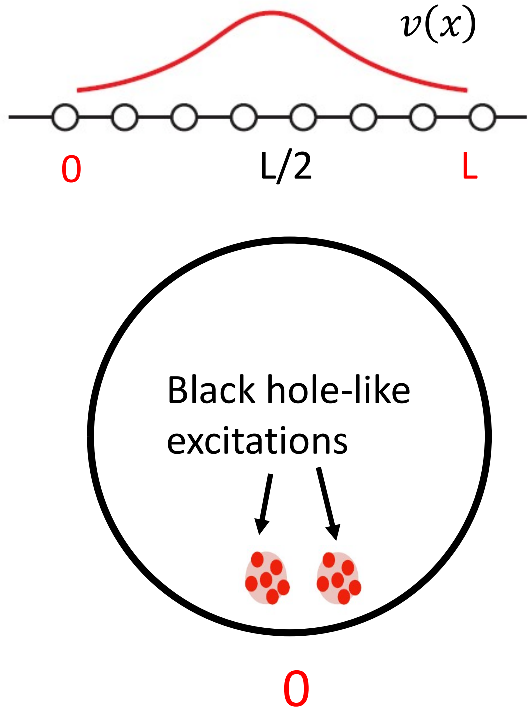




Quasiparticle Picture For Free Fermions

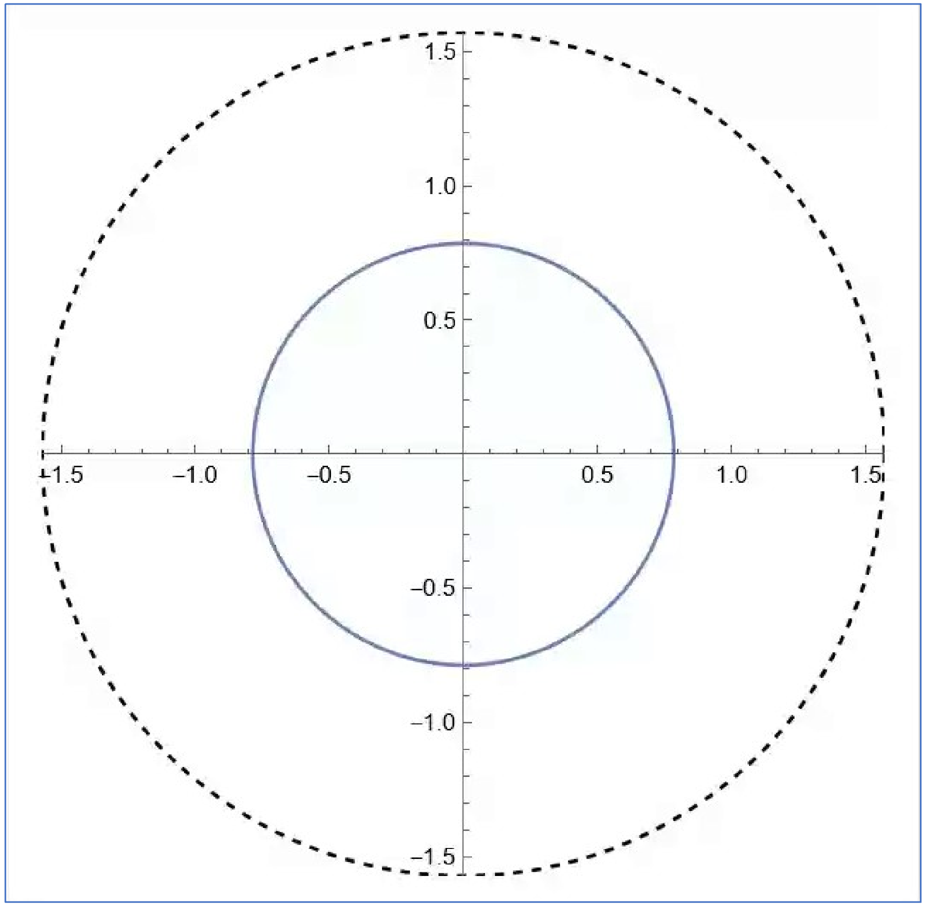
- Quasiparticles conserved so density obeys continuity equation





Gravitational dual for Holographic CFTs

- In holographic systems, the bulk horizon gets deformed with two spikes appearing
- For SSD, when $t \to \infty$, the spikes merge and touch the asymptotic boundary

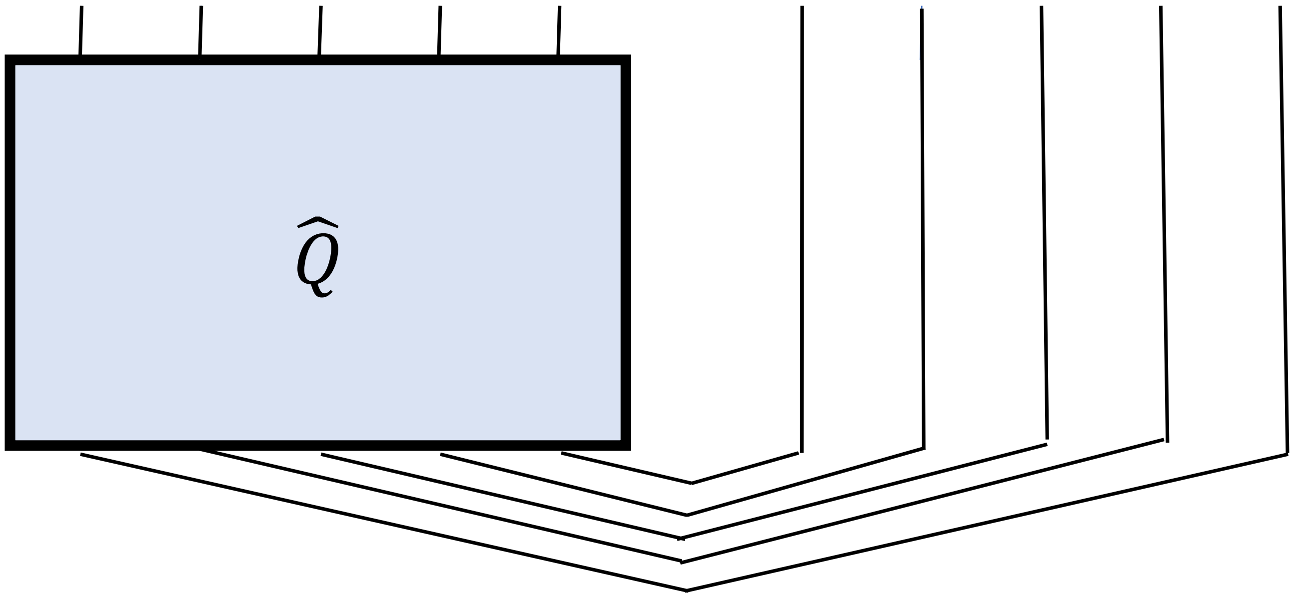


Operator Entanglement

- Think of operators as states in the operator Hilbert space

$$\hat{Q} = \sum_{n,m} \langle n|\hat{Q}|m\rangle |n\rangle\langle m| \to |\hat{Q}\rangle = \sum_{n,m} \langle n|\hat{Q}|m\rangle |n\rangle \otimes |m\rangle^*$$

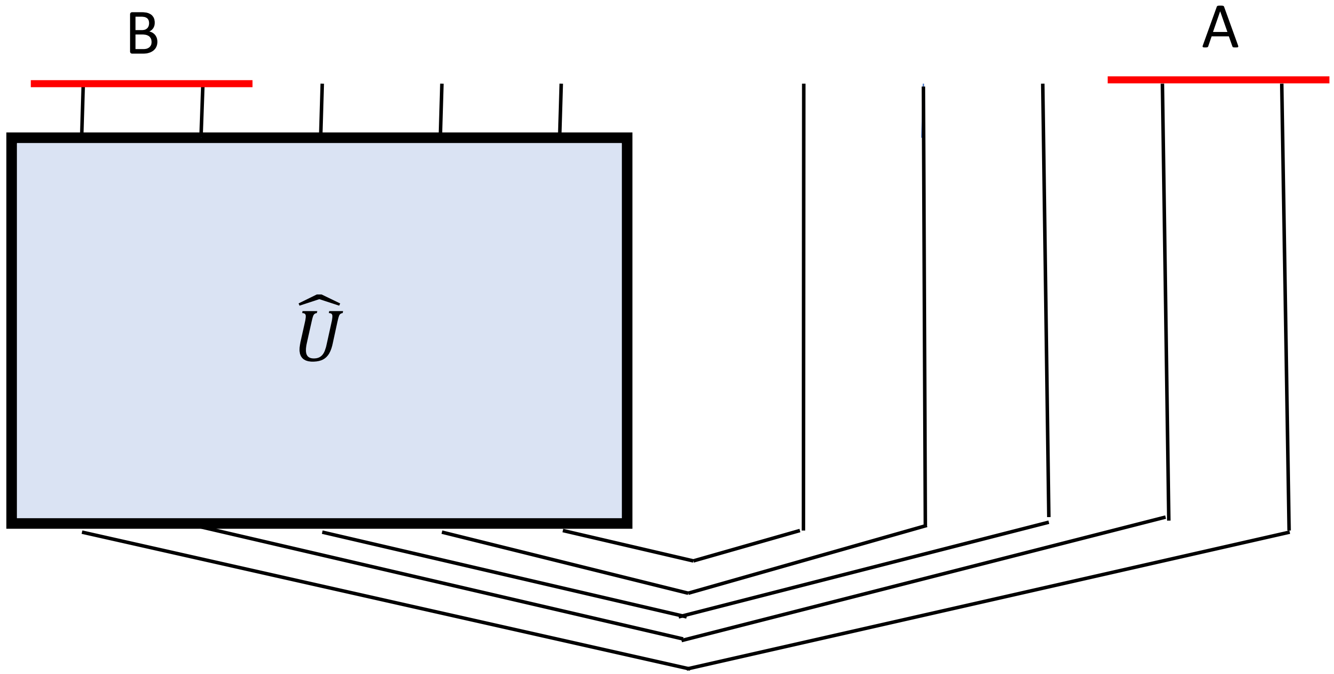
$$= (\hat{Q} \otimes \mathbb{I}) \textstyle\sum_m |m\rangle \otimes |m\rangle^*$$



Operator Entanglement

- Study the entanglement entropy of these states.

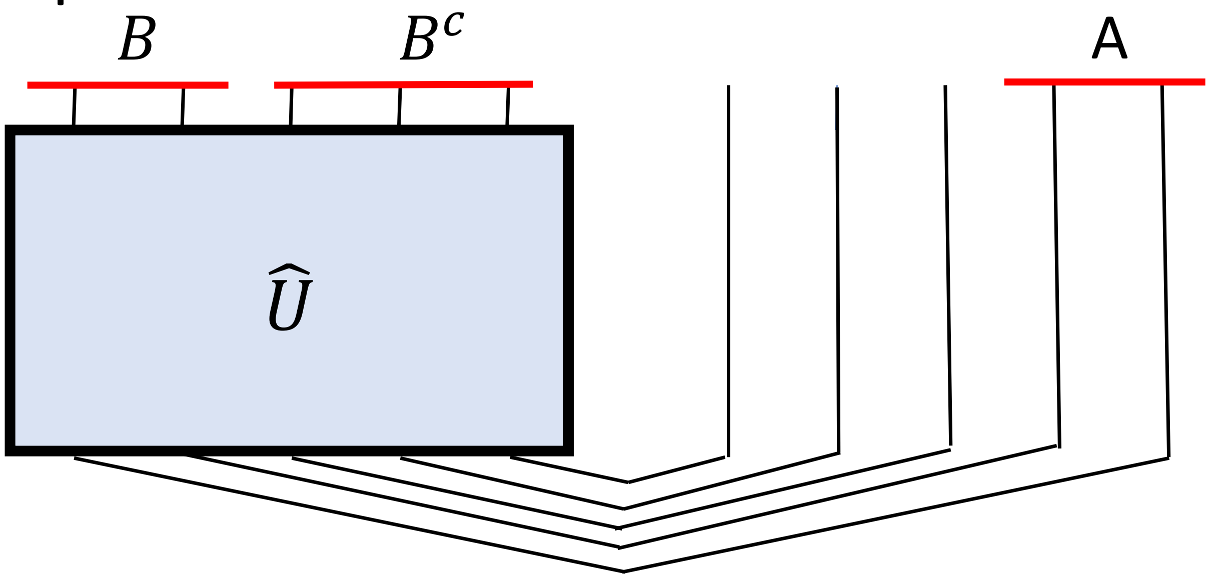
(Zanardi, Prosen, Pižorn, …)



- Bipartite Operator Mutual Information (BOMI) measures the correlation between subregions A and B

$$I(A,B\ ) = S_A + S_B - S_{A\cup B} \geq 0$$

Tripartite Operator Mutual Information (TOMI)



- $I_3(A, B, B^c) = I(A, B) + I(A, B^c) - I(A, B \cup B^c)$

  Local Local Non-local

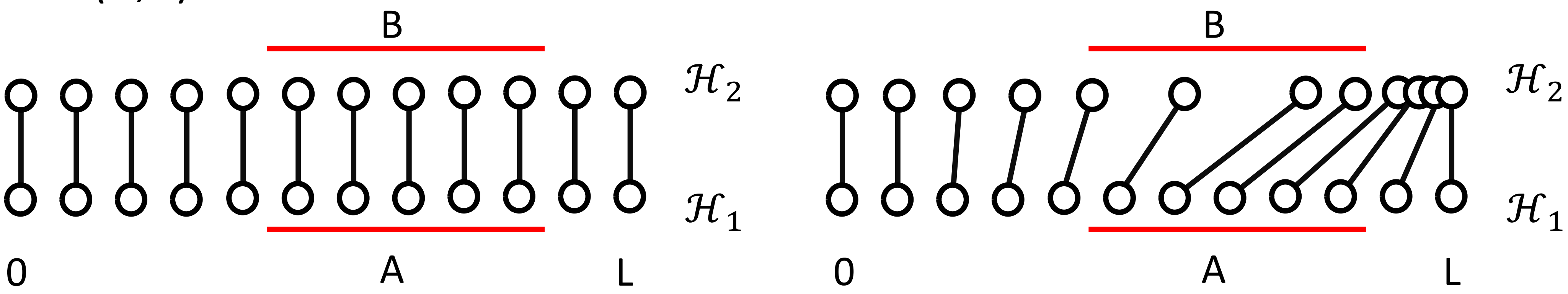
- Scrambling is the delocalization of information
- Non-local $>$ Local $\Rightarrow I_3(A, B, B^c) < 0$
- We will use tripartite mutual information to study information scrambling (Hosur et. al.)

Quasiparticle Picture For Free Fermions

- Generic Operator state:

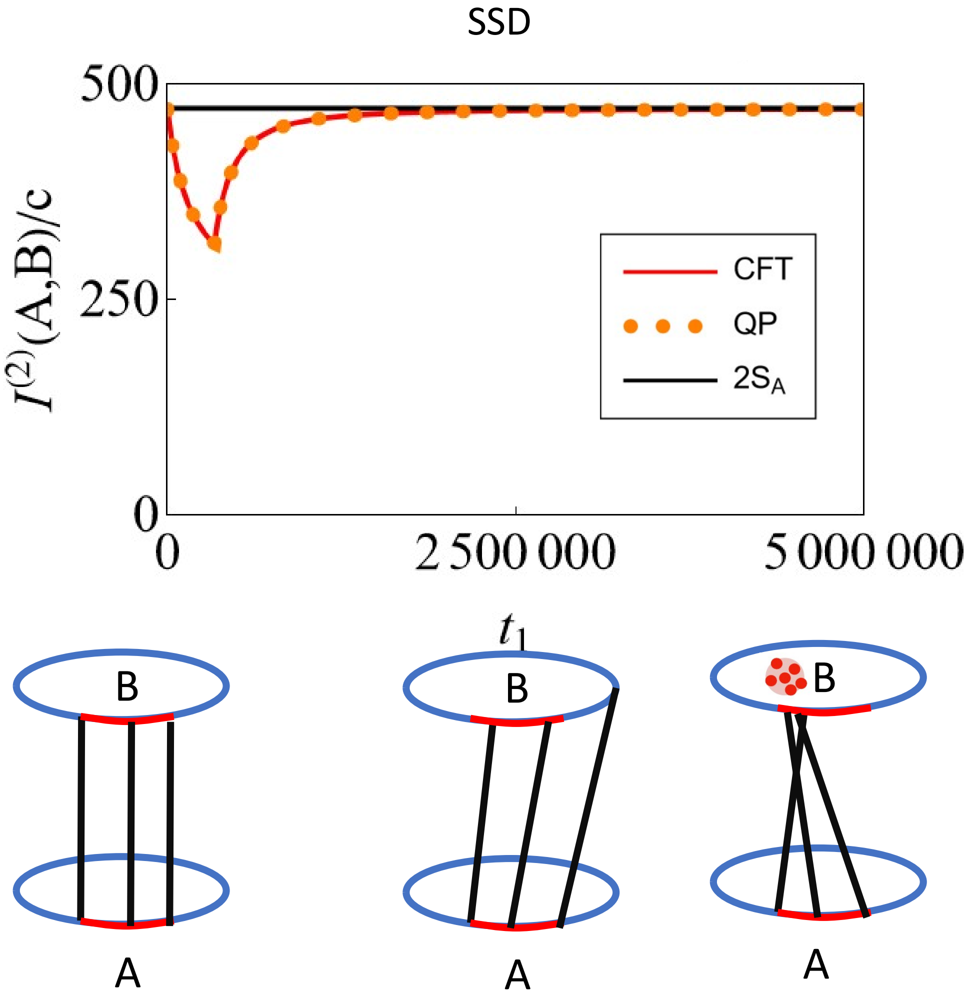
$$|U\rangle = U \otimes \mathbb{I} \prod_x |Bell\rangle_x$$

- Operator Entanglement for free fermions well-described by motion of bell pairs
- One end of each Bell pair moves with speed $\mathrm{f(x)} = 2\sin^2\left(\frac{\pi x}{L}\right)$
- I(A,B) $\sim$ No. of Bell Pairs shared between A and B

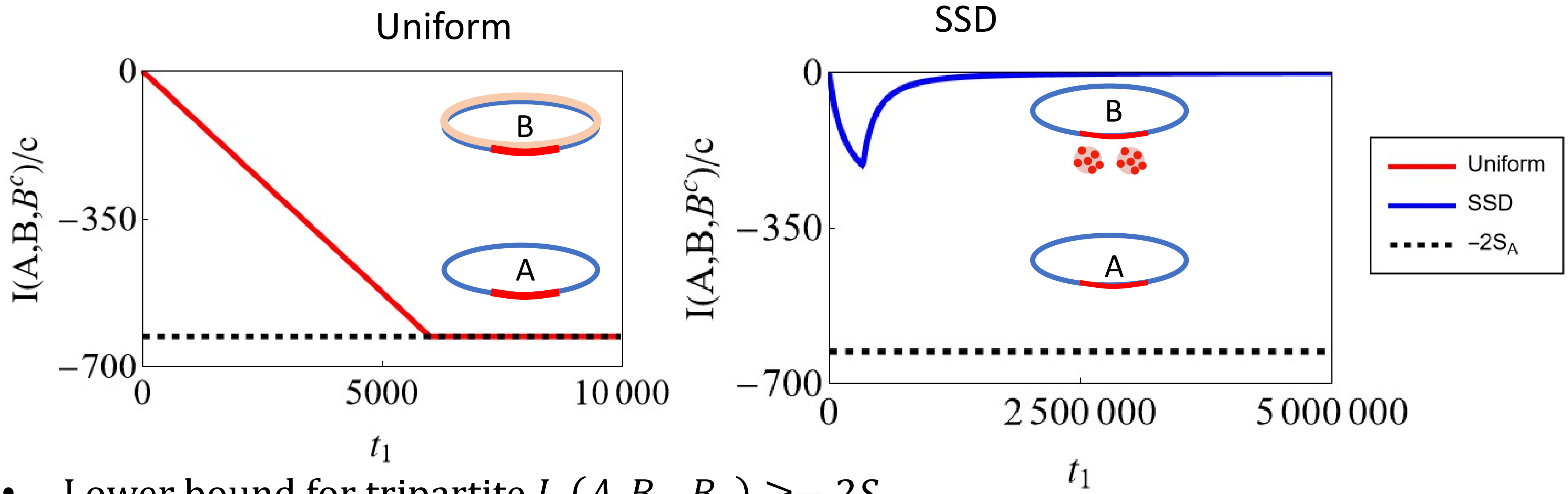


Operator Mutual Information in Free Fermions

- A and B centered about origin.
- $I_3(A, B, B^c) = 0$



Information Scrambling in Holographic CFTs



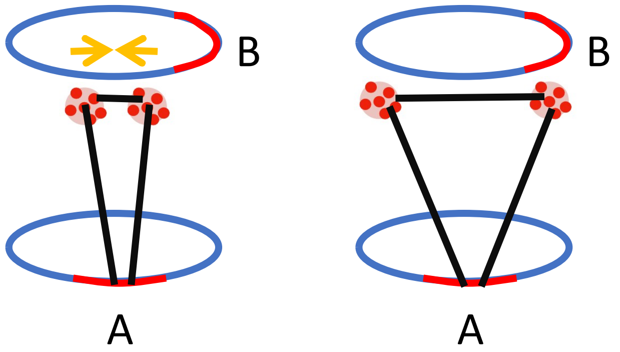
- Lower bound for tripartite $I_3(A, B_1, B_2) \geq -\, 2S_A$
- Uniform Hamiltonian ⇒ saturates at most negative value ⇒ maximal information scrambling
- SSD Hamiltonian ⇒ Information scrambling eliminated at late times
- Consistent with formation of localized black hole-like excitations

Genuine Tripartite Mutual Information

$$U = e^{-i\,H_0 t_0} e^{-i\,H_{SSD} t_1}$$

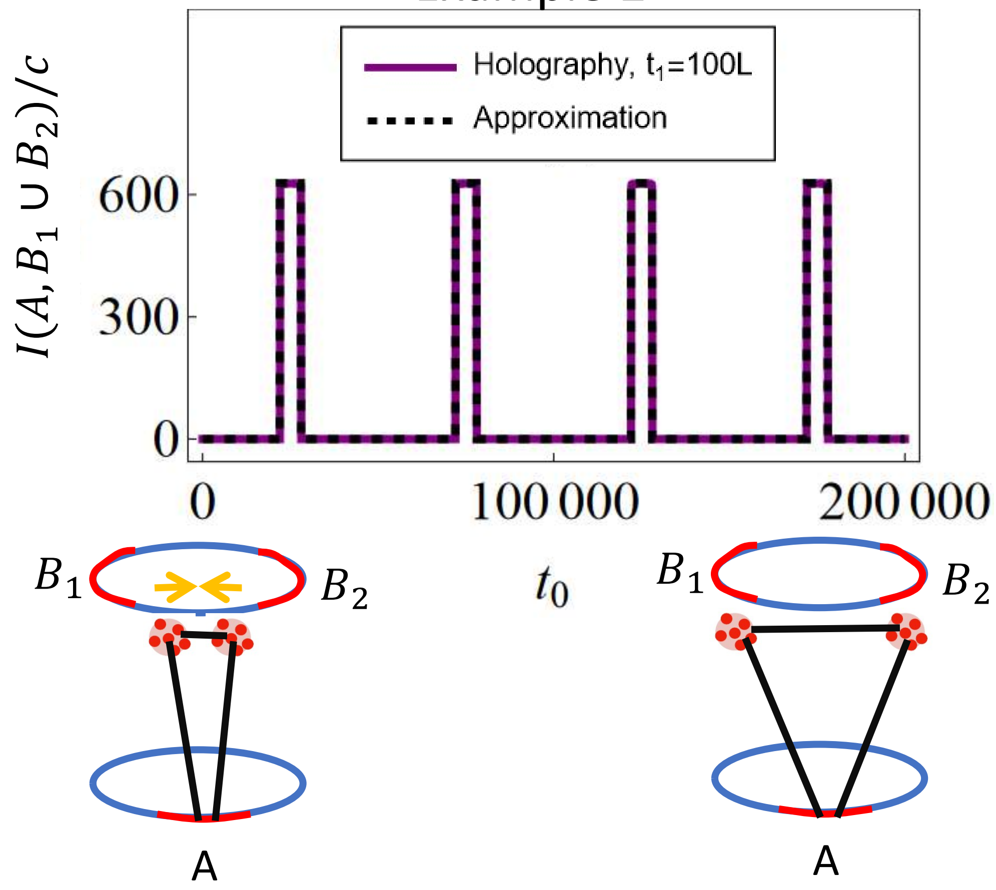
- Evolve with SSD first to create a black hole-like excitation then evolve with uniform Hamiltonian
- Mutual information non-zero only if subsystem B contains both black hole-like excitations

Example 1



$$I(A,B) = 0$$

Example 2

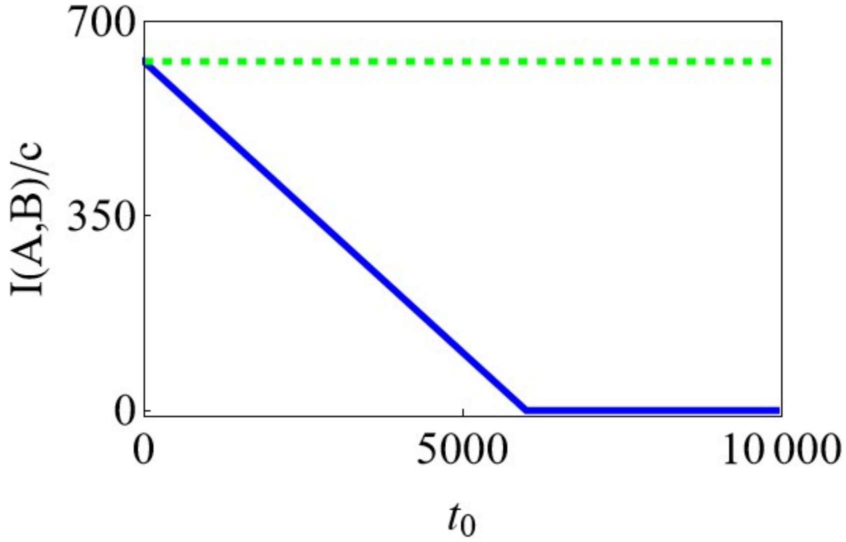


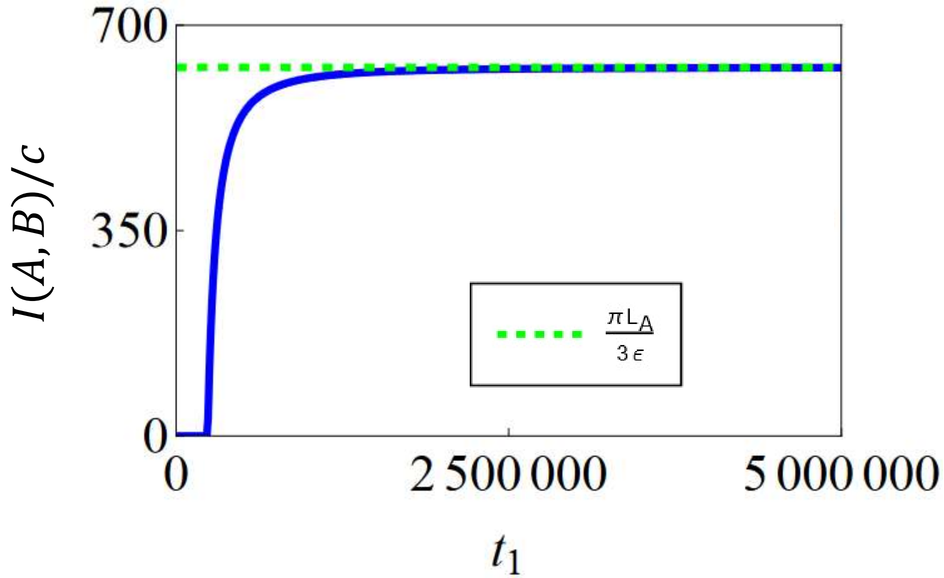
Conclusion

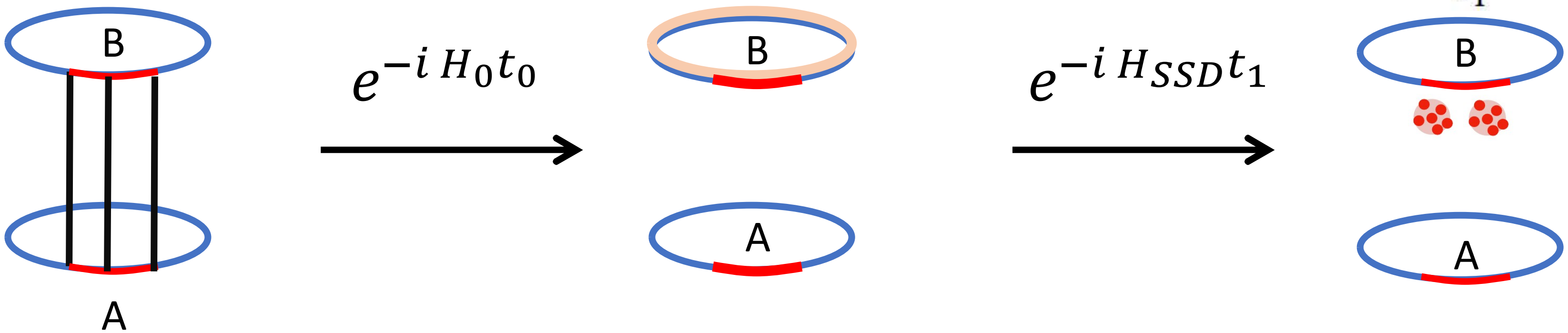
- Studied inhomogeneous quenches in free fermion and holographic CFTs
- Information gets concentrated around a fixed point, cooling the rest of the system.
- Genuine tripartite mutual information produced in holographic CFTs
- Future direction: Study other systems, other driving protocols, other physical quantities.

Recovery of Quantum Information

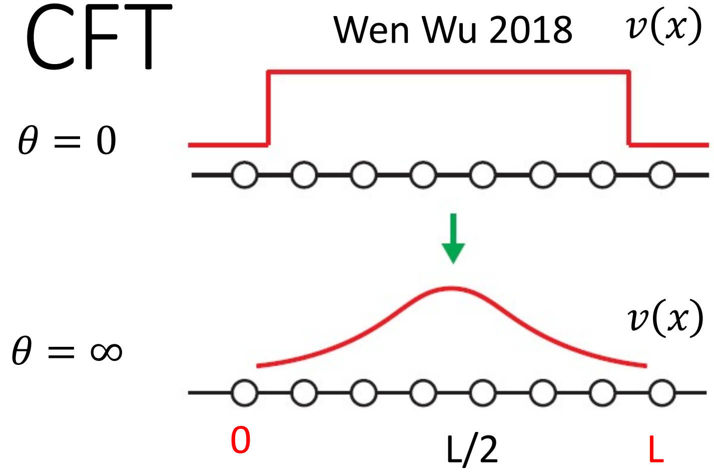
$$U = e^{-i\,H_{SSD}t_1}e^{-i\,H_0 t_0}$$







Inhomogeneous Quench in 1+1d CFT



- Let $h(x)$ be the energy density so that

$$H_0 = \int_0^L dx\, h(x)$$

- The spatially inhomogeneous Hamiltonian:

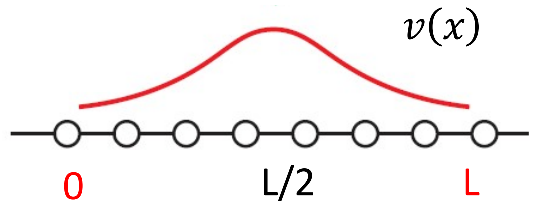
$$H_\theta = \int_0^L dx\, v(x)\, h(x) \text{ where } v(x) = 1 - \tanh 2\theta \cos \frac{2\pi x}{L}$$

- The sine-squared deformation (SSD) limit is

$$H_{\theta\to\infty} = \int_0^L dx\, 2\sin^2\left(\frac{\pi x}{L}\right)\, h(x) \equiv H_{SSD}$$

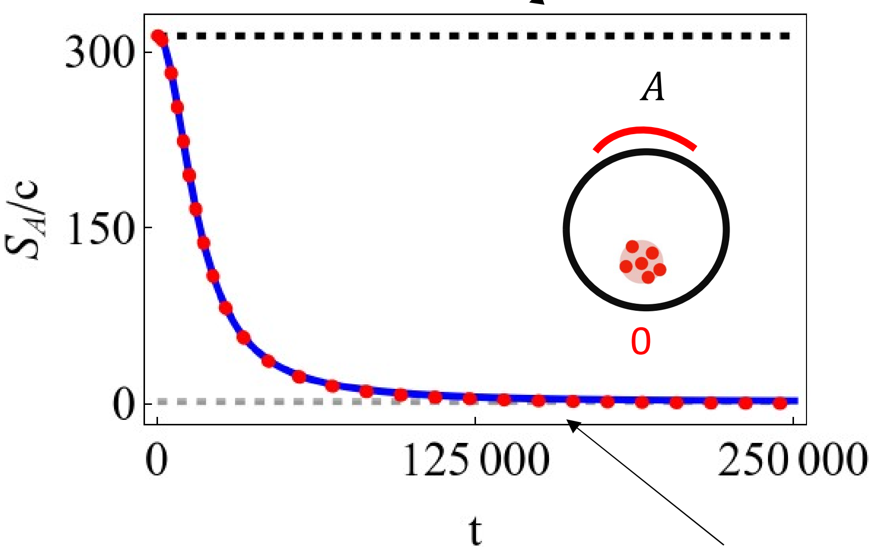
- SSD envelope vanishes at 0 and maximum at $\frac{L}{2}$
- $\theta = 0$ (uniform) $\rightarrow \theta = \infty$ (sine-squared deformed)

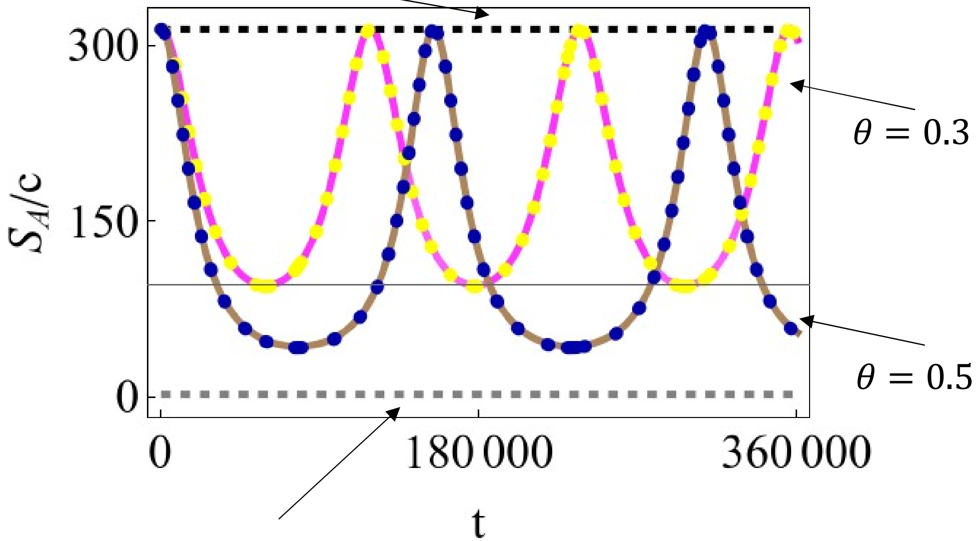
Entanglement Entropy during Quench



Thermal Entropy $S_A/c = \frac{\pi L_A}{6\epsilon}$

$\theta = \infty$

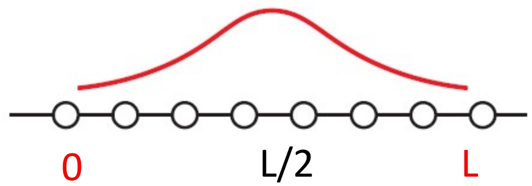




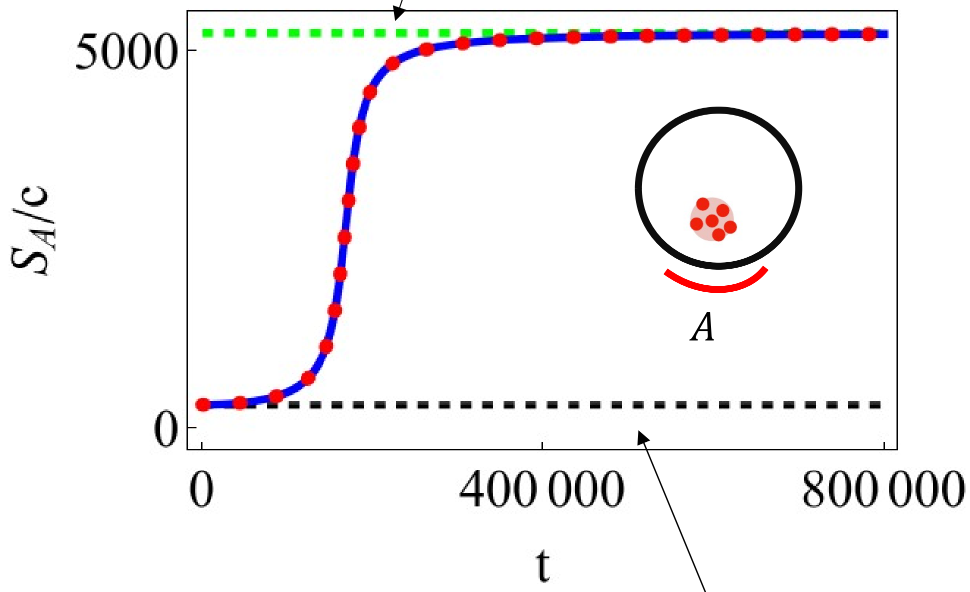
Ground-state value $S_A/c = \frac{1}{3}\ln\left[\frac{L}{\pi}\sin\frac{\pi L_A}{L}\right]$

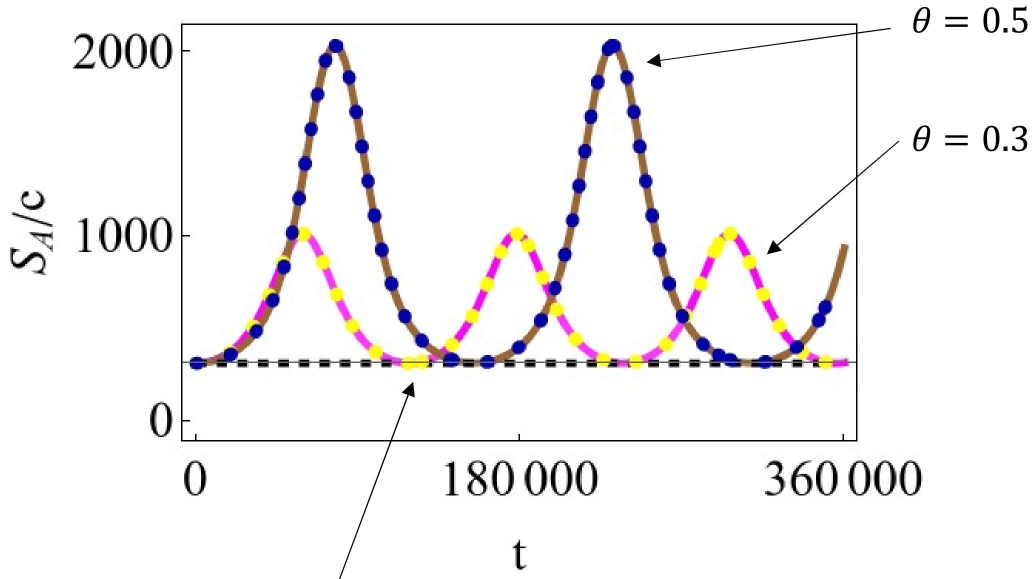
- Holographic CFT and free fermion CFT similar
- In the SSD limit, when subsystem is away from origin, entanglement entropy goes from thermal entropy of subsystem to ground state entanglement entropy
- For finite θ, observe oscillations with period $L\cosh 2\theta$

Entanglement Entropy during Quench



$\theta = \infty$ Total entropy $S_{tot}/c = \frac{\pi L}{6\epsilon}$





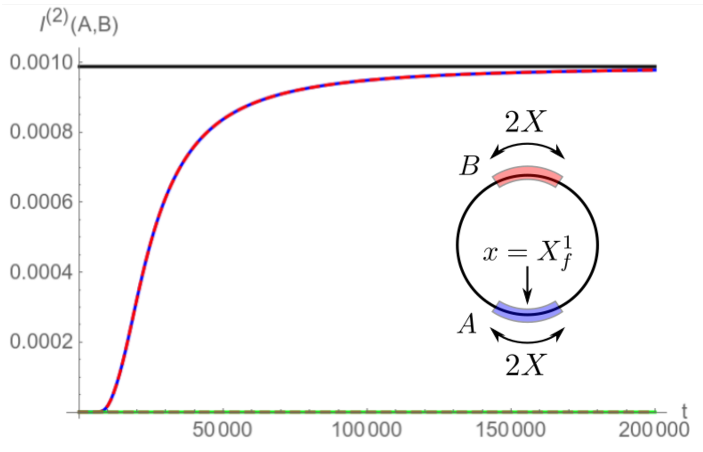
Thermal entropy $S_A/c = \frac{\pi L_A}{6\epsilon}$

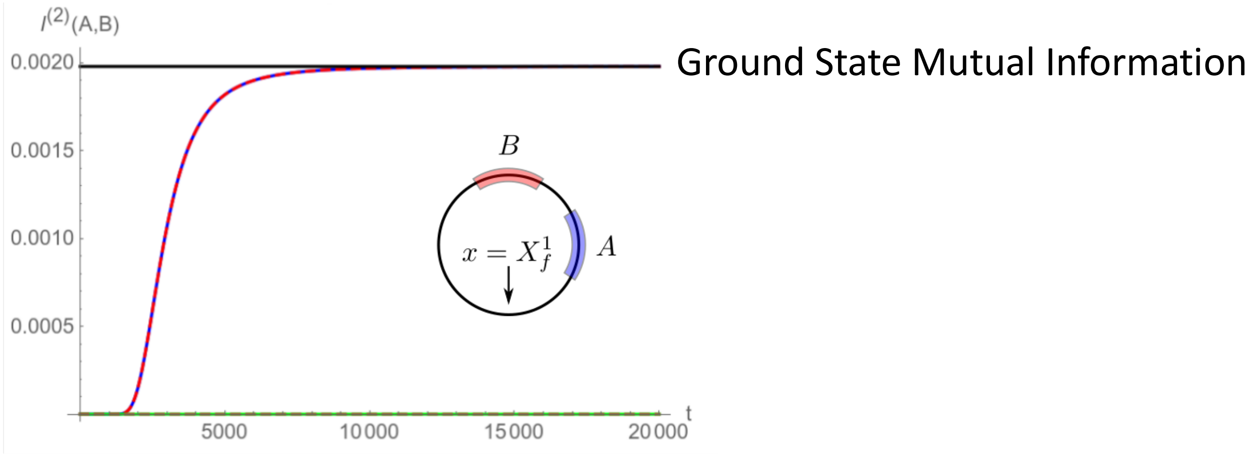
When subsystem contains the origin, entanglement entropy goes from thermal value to thermal entropy of the total system

Mutual Information

- The mutual information $I(A,B) = S_A + S_B - S_{AB}$ approaches the ground state value

Free Fermion CFT

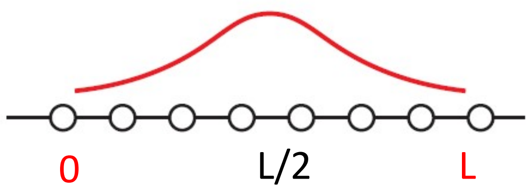




- Mutual information for holographic CFTs also approaches the ground state value

Inhomogeneous Quench of Thermal State

- Quench the uniform thermal state with Möbius Hamiltonian in 1+1d CFT
- $\rho(t) = e^{-iH_\theta t} \frac{e^{-\beta H_0}}{Z} e^{iH_\theta t} = Z^{-1} e^{-\beta H_0(t)}$ with $Z = Tr\, e^{-\beta H_0}$
- $H_0(t)$ and hence $\rho(t)$ is periodic with period $L\cosh 2\theta$
- For $x \neq 0, \lim_{\theta\to\infty} \rho(t) \sim e^{-\frac{\beta\pi^2 t^2}{L^2} H_{SSD}}$
- If the ground state of H_{SSD} is the same as H_0, $\rho(t)$ in the SSD limit at late times is approximately the uniform ground state
- Away from $x = 0$, system "reverse thermalized"

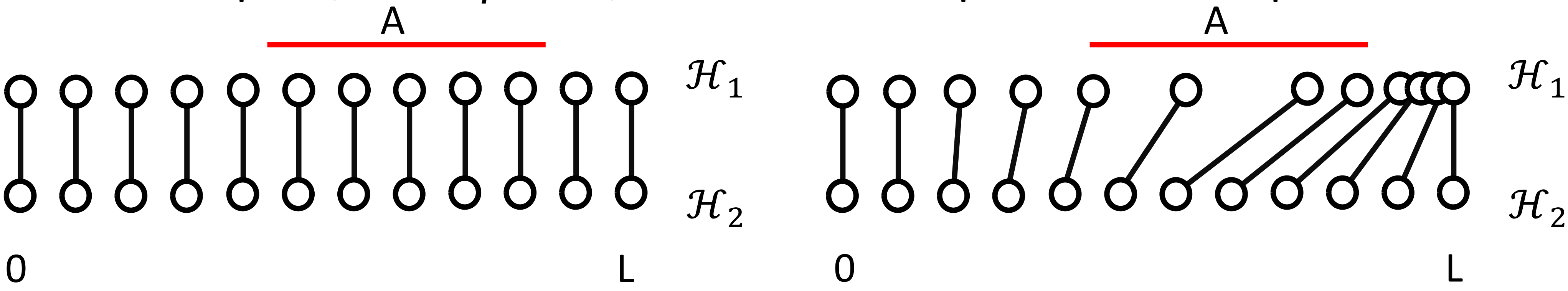


Quasiparticle Picture For Free Fermions

- Purify the thermal state to thermofield double state

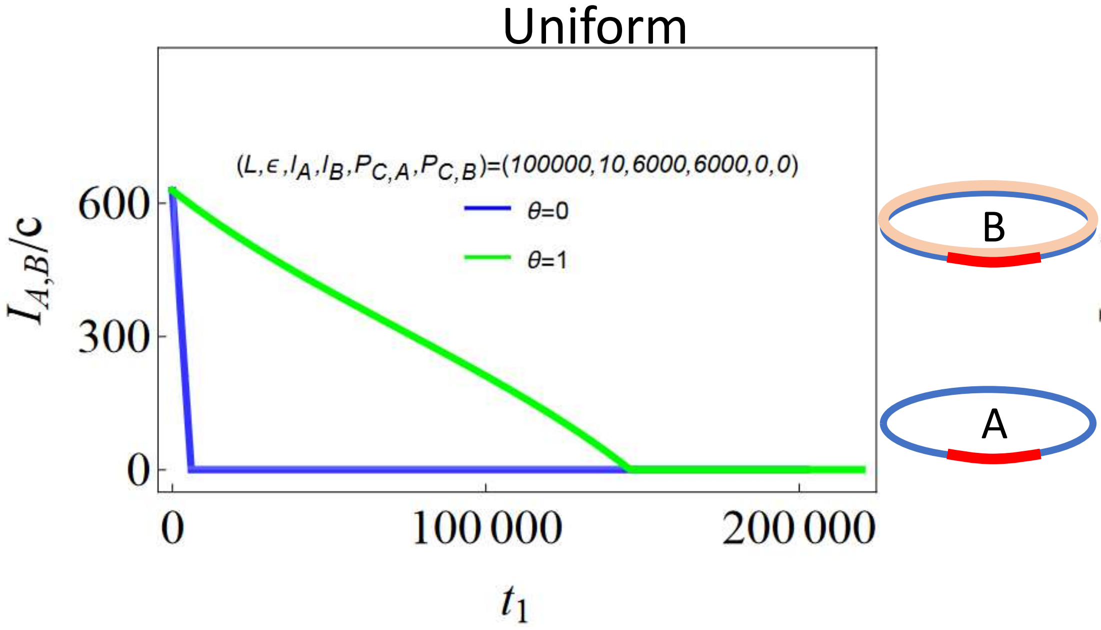
$$|TFD\rangle \sim \sum_E e^{-\beta E/2} |E\rangle_{\mathcal{H}_1} |E\rangle_{\mathcal{H}_2} \Rightarrow \; e^{-\beta H} = Tr_{\mathcal{H}_2} |TFD\rangle\langle TFD|$$

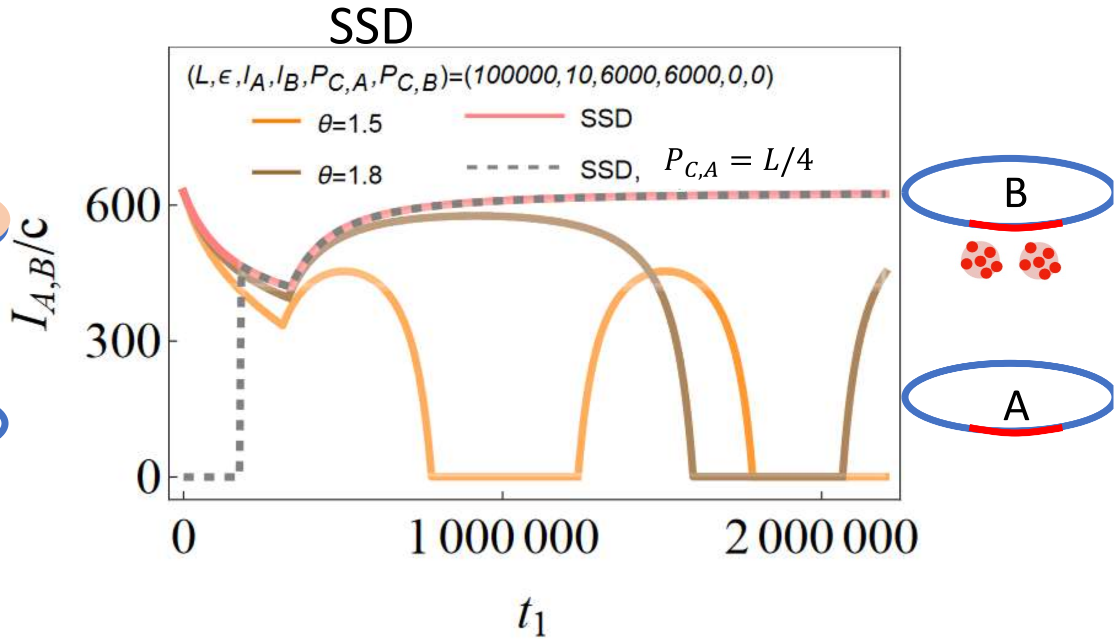
- In real space, when $\beta \to 0$, TFD looks like a product of Bell pairs



- One end of each Bell pair moves with speed $\mathrm{f(x)} = 1 - \tanh 2\theta \cos \frac{2\,\pi\,x}{L}$
- Entanglement Entropy $\sim$ No. of Bell Pairs in A

Information Scrambling in Holographic CFTs

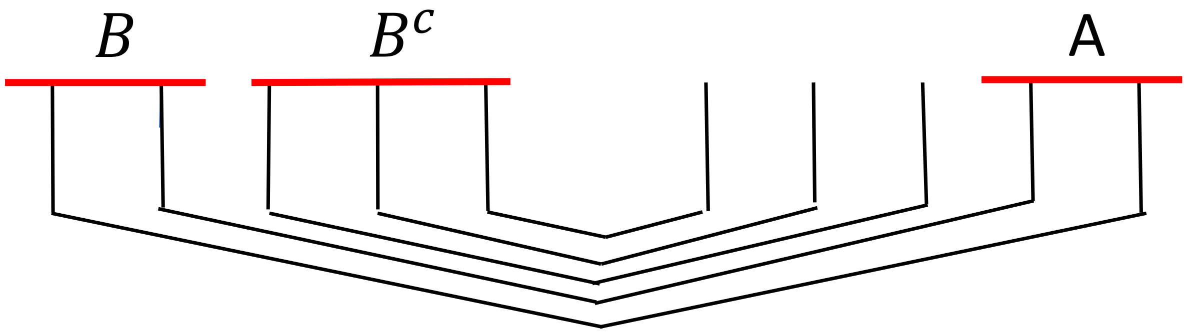




- Lower bound for tripartite $I_3(A, B_1, B_2) \geq -2S_A$
- Uniform Hamiltonian ⇒ saturates at most negative value ⇒ maximal information scrambling
- SSD Hamiltonian ⇒ Information scrambling eliminated at late times
- Consistent with formation of localized black hole-like excitations

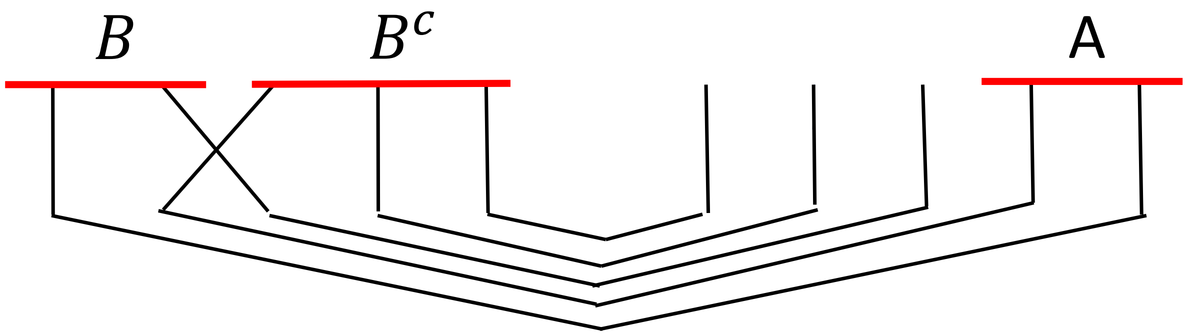
Simple Examples

$\hat{U} = \mathbb{I}$



$$I_3(A, B, B^c) = 0$$

$\hat{U} = SWAP_{2,3}$



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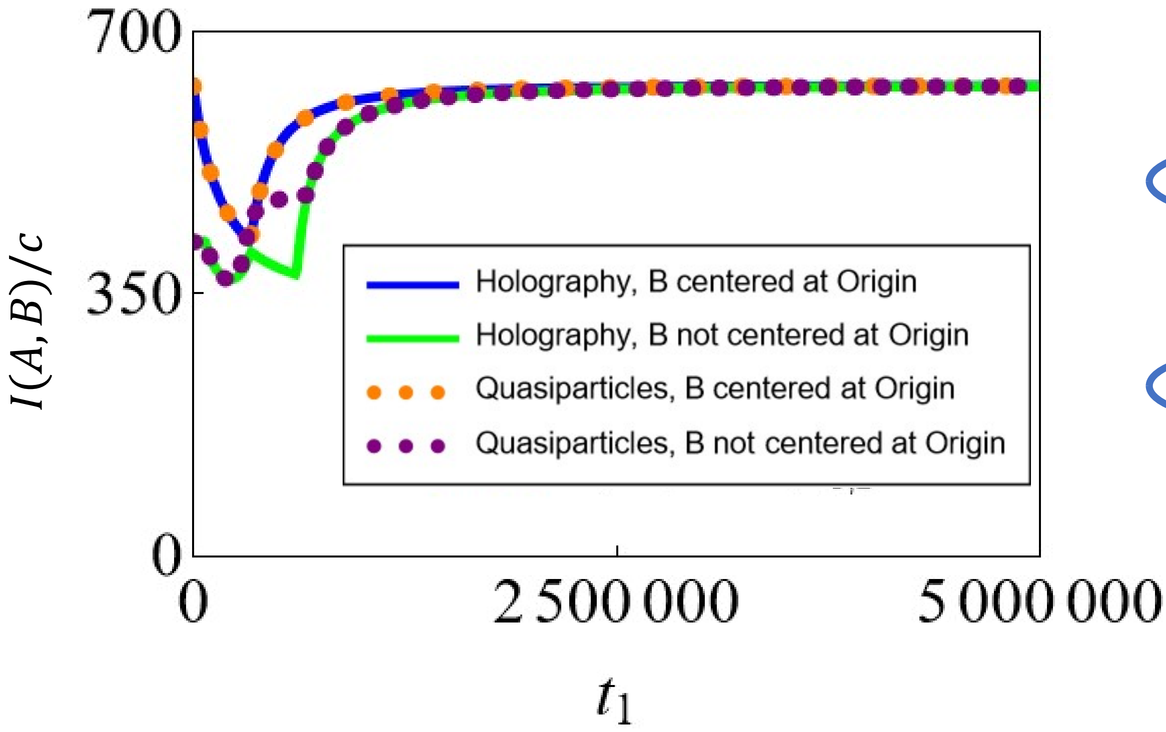
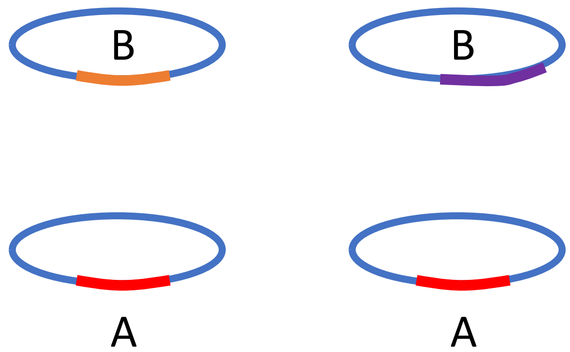
Operator Mutual Information in Holographic CFTs



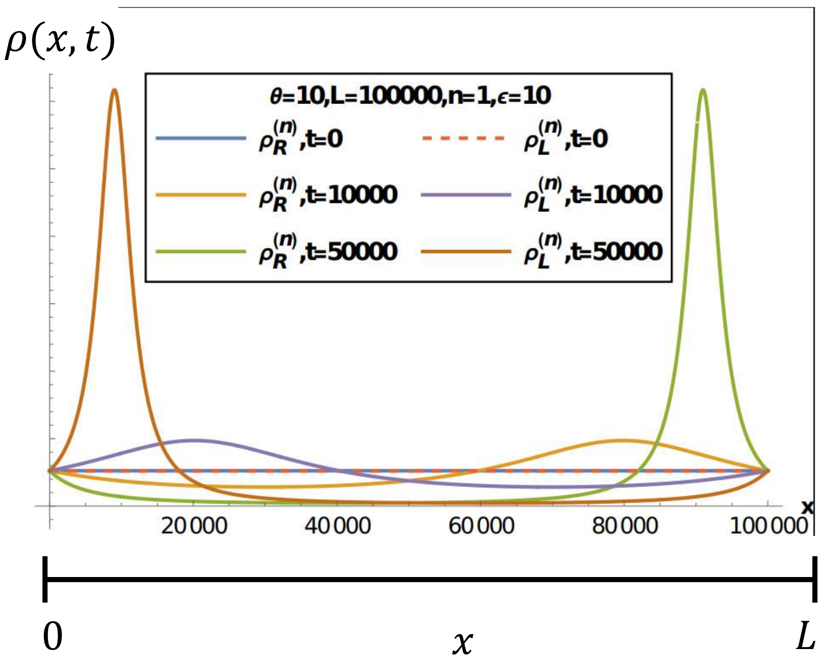


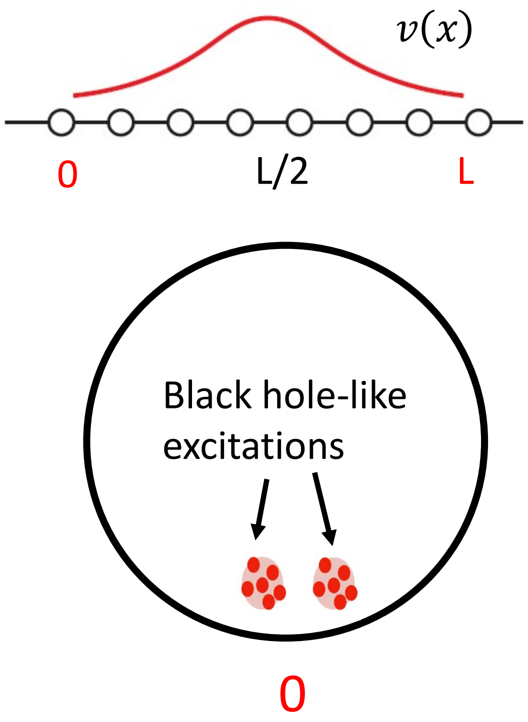
Quasiparticle description does not work well for Holographic CFTs.

Quasiparticle Picture For Free Fermions

- Quasiparticles conserved so density obeys continuity equation

Inhomogeneous





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